TO SOLVE A POLYNOMIAL INEQUALITY

- Put everything on the left hand side, so that only zero is on the right hand side.
- Let $f(x)$ be the polynomial function on the left hand side. Completely factor $f(x)$.
- Make a sign chart for f.
  - Mark each zero on a line, in ascending from left to right.
  - Start on the far right. If the leading coefficient is positive, then this interval is positive. If the leading coefficient is negative, then this interval is negative.
  - Proceed from right to left; the sign can only change at a zero. If the multiplicity of the zero is odd, the sign changes. If the multiplicity of the zero is even, the sign does not change.
- Use the relational operator ($\leq$, $\geq$, $<$, or $>$) to determine the interval(s) of solution. Write the intervals, connecting them with union symbols.
  - $\leq$ Intervals where $f(x)$ is negative. Included endpoints $\Rightarrow$ use brackets.
  - $\geq$ Intervals where $f(x)$ is positive. Included endpoints $\Rightarrow$ use brackets.
  - $<$ Intervals where $f(x)$ is negative. Excluded endpoints $\Rightarrow$ use parens.
  - $>$ Intervals where $f(x)$ is positive. Excluded endpoints $\Rightarrow$ use parens.

Example 1. Solve $x^3 + 10 \geq 2x^2 + 5x$.

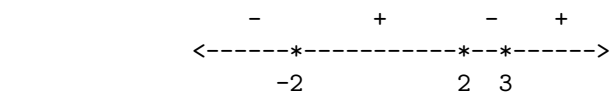
Solution. Put everything on the left hand side. We get

$$x^3 - 3x^2 - 5x + 15 \geq 0.$$

Let $f(x) = x^3 - 3x^2 - 4x + 12$. Now

$$f(x) = x^2(x-3) - 4(x-3) = (x^2-4)(x-3) = (x+2)(x-2)(x-3).$$

The sign chart is



The solution uses bracket because the relational operator is greater than or equal two, so the zeros are also solutions. The solution set is

$$[-2, 2] \cup [3, \infty).$$

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